TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 131

VARIATION IN THE NUMBER OF REVOLUTIONS OF AIR PROPELLERS.

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From Technische Berichte, Volume III, Part 2.

March, 1923.

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If an air propeller is to drive the airplane at its maximum
flying speed, it must run at the number of revolutions in which
the engine develops its maximum power. It is at present quite im-
possible to calculate mathematically these revolutions in advance,
and model experiments also will hardly meet the accuracy at pres-
ent required, especially in new construction, as all variable el-
ements cannot be taken into consideration in these experiments.
Even the greatest mathematical accuracy of ± 2 to 3% causes,
owing to the high revolution speeds of aircraft engines, an appre-
ciable variation of some ± 30 to 50 R.P.M. It is desirable to
eliminate or reduce even this small fluctuation.

Speaking broadly, it is only possible to increase the revolu-
tions per minute by diminishing either the diameter, or the width
of blade. It is far more difficult to reduce the speed and usu-
ally the only possible method is to increase the pitch, which gen-
erally entails the manufacture of a new air propeller; but even in
such a case, it is necessary to be able to designate with a cer-
tain degree of accuracy, which dimensions of the original model
must be altered, in order to fulfill the conditions intended.

Everyone connected with airplane and air propeller construc-

tion will know the trouble, time, and money which is spent in test-
ing propellers. The present period especially, witnessing the
transition to the supercharged engine, makes great demands on the
designers and pilots, and many delays in the delivery of important
airplanes would have been obviated, if the adaptation of the air
propeller to the engine had been easier.

The present paper is intended to be of some help in this di-
rection, as the simple formulas given permit advanced calculation
to be made of the variation of propeller speed with variation of one
or more dimensions of the propeller, and allow the necessary cor-
rections to be applied.

The connection between the engine power HP and the propeller
speed N is given by the equation

$$HP = (K\,D)^2\,(N\,p)^3 \tag{1}*$$

Where: D = diameter of propeller in m.

p = pitch   "   "   " " " "

K = A coefficient varying with the airplane speed, but con-
stant at similar speeds and with similar propellers. Assuming the
variations in speed to be small (not exceeding $\pm 10\%$), K may be
assumed constant.

Influence of variations in engine power.

It can often be observed in new designs of engines that the
actual output does not correspond with the rated horsepower; but

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* According to Eiffel, Gumbel and Schaffran, the required torque is
$Q = c_2\,N^2 D^2 p^3$; further $\omega = \dfrac{\pi\,N}{30}$ therefore: $\dfrac{Q\,\omega}{75} = HP = \dfrac{c_2\,\pi}{75} D^2\,N^3\,p^3$
or, $HP = (K\,D)^3\,(N\,p)^3$ as above).

usually somewhat exceeds it. In consequence, the propeller designed for the rated horsepower is operated at too high a speed. The following question therefore must be answered: What increase in the R.P.M. is produced by the increase in engine output?

If HP varies as $\pm \Delta$ HP, N varies as $\pm \Delta$ N, and equation (1) becomes:

$$HP \pm \Delta HP = (K D)^2 (N \pm \Delta N)^3 \bar{p}^3$$
$$= (K D)^2 (N p)^3 (1 \pm \frac{3 \Delta N}{N})$$

if the higher powers of $\frac{\Delta N}{N}$ are disregarded. If HP is substituted for $(K D)^2 (Np)^3$, then HP $\pm \Delta$ HP = HP $(1 \pm 3 \frac{\Delta N}{N})$ or

$$\Delta N = \pm \frac{N}{3} \cdot \frac{\Delta HP}{HP} \ldots \qquad (2)$$

(the same result is obtained if $\left(\frac{dHP}{dN} \right)$ is obtained from equation 1.)

Example: A 240 HP engine develops actually 50 HP more. The propeller, geared down in a 1 : 2 ratio makes on the testing stand N = 650 R.P.M., owing to the increased engine output.

$\Delta N = + \frac{650}{3} \frac{50}{240} = + 45$ R.P.M. for the propeller and +90 R.P.M. for the engine, which corresponds with practice.

Influence of altered diameter.

The usual method of increasing the speed, is in shortening the blade and so long as the amount removed is not excessive this practice is not objectionable. Increasing the diameter when the speed is too high, often presents difficulty on account of the height of the landing gear. What is now the fluctuation of the R.P.M., N when the diameter D is varied by $\pm \Delta D$? It must be remembered that each variation in the speed occasions also a

variation in the engine output, HP.

Assuming that the engine output fluctuates, within the narrow limits set, proportionately to the R.P.M.; if N varies by $\pm \Delta N$, then HP becomes $HP \frac{N + \Delta N}{N}$; and equation (1) becomes:

$$HP \frac{N \mp \Delta N}{N} = [K (D \pm \Delta D)]^2 \ [(N \mp \Delta N) \ p]^3 ;$$

therefore

$$\frac{N \mp \Delta N}{N \mp 3\Delta N} = 1 \ \pm 2 \frac{\Delta D}{D} \tag{3}$$

Example: What increase is necessary in the diameter of the propeller in the first example, in order to reduce its speed 45 R.P.M.? The original diameter was 4.10 m (13.45 ft).

$$\frac{695 - 45}{695 - 135} = \frac{2\Delta D}{4.1} + 1.$$

$\Delta D = 0.33$ m (1.08 ft) so that the new diameter is:

$$D + \Delta D = 4.43 \text{ m } (14.53 \text{ ft})$$

Another propeller having a diameter of 2.84 m (9.32 ft) runs at 1320 R.P.M. on the stand; it is necessary to run it at 40 R.P.M. more. What reduction in diameter is necessary?

$$\frac{1320 + 40}{1320 + 120} = 1 - \frac{2}{2.84} \ \Delta D$$

The new diameter is $D = 2.84 - 0.08 = 2.76$ m (9.06 ft).

If the fluctuation of speed is over 10%, it is advisable to calculate the reduction for a 10% increase, then test the propeller on the stand, and find new a diameter by means of a second approx-

imation based on the result of this test.

Influence of alteration of pitch.

A completed propeller allows of only a small alteration of pitch - by pitch is meant the theoretical pitch of the working face - as this always weakens the blade, and this cannot be disregarded if the propeller has been correctly designed. In the first instance it is also, in many cases, highly undesirable to vary the pitch, in order to arrive at the correct engine speed, as the speed of the airplane is thereby affected. If such is decided upon, however - perhaps because it is desired to see whether the speed of the airplane will increase with the altered pitch - the expected variation in the R.P.M. must be calculated, in order that it may be taken account of when making the alterations.

From equation (1) we have:

$$HP \; \frac{N \mp \Delta N}{N} = (K \; D)^2 \; (N \mp \Delta N)^3 \; (p \pm \Delta p)^3$$

Thence

$$\frac{N \mp \Delta N}{N \mp 3 \, \Delta N} = 1 \pm 3 \, \frac{\Delta p}{p} \tag{4}$$

Example: A propeller of 2.9 m (9.51 ft) pitch makes approximately 100 R.P.M. in excess of those required. The pitch can be altered about 0.1 m to 3.0 m (3.28 to 9.84 ft). What benefit is obtained thereby? Is it better, in order to save time, to install a new propeller instead? The speed is $N = 800$ R.P.M.

$$\frac{800 - \Delta N}{800 - 3 \, \Delta N} = 1 + \frac{10}{290}$$

$$\Delta N \cong 25 \text{ R.P.M.}$$

The engine working on a 2 : 1 gear, developed on test only 50 R.P.M. less, so that the time utilized for altering the pitch had been lost.

Influence of variation of blade width.

The blade with C does not occur in equation (1), as in similar propellers - this width alters with the diameter, and its influence is already included in the factor K. As the blade width is small in relation to the diameter, the alteration of this dimension has a far greater effect on the revolution speed than the alteration of the diameter or the pitch; widening or narrowing the blade is often a far quicker way of attaining the same result.

An easy approximation to the correct value of blade width is arrived at if we assume the blade divided into strips of equal area ΔS, having equal breadth Δc at the center. The power transmitted by the propeller is considered to be distributed uniformly over the width of the blade, and each strip takes up the share $\Delta HP = HP \frac{\Delta c}{c}$ of the engine output, HP.

$$\frac{\Delta HP}{HP} = \frac{\Delta c}{c}$$

then

$$N \pm \Delta N = N \frac{c}{c \mp \Delta c} \pm \frac{N}{3} \frac{\Delta c}{c}$$

$$\pm \frac{\Delta N}{N} = \frac{c}{c \mp \Delta c} \pm \frac{\Delta c}{3\ c} - 1 \tag{5}$$

Example: A propeller with an average width of blade of 0.22 m (.72 ft) runs at 780 R.P.M.

It is desired to ascertain what increase of speed would be brought about by decreasing the width 0.01 m (.033 ft) = $\frac{1}{22}$ c

$$+ \frac{\Delta N}{780} = \frac{0.22}{0.21} + \frac{0.01}{0.66} - 1$$

$$\Delta N = 780 \times 0.0627 \cong 49 \text{ R.P.M.}$$

Such, approximately, was the observed result.

Cooperation of several variations.

According to the law for the summation of forces, we have:

$$\Sigma \Delta N = \pm \Delta N_Q \pm \Delta N_D \pm \Delta N_P \pm \Delta N_c \qquad (6)$$

Where the suffixes are self-explanatory. A frequent exception is that $\Sigma \Delta N = 0$ i.e. the various alterations neutralize each other.

Propeller construction has steadily progressed since the outbreak of the war, as the designer of the propeller has found it necessary to keep pace with the airplane and engine manufacturers. There is very little time available for the testing of the first experimental propellers on a new airplane, and the constructorn is obliged to rapidly design new forms on the basis of available data alone, without lengthy calculations. The formulas given above should satisfy this need, but if used absolutely without practical data, will also prove unsatisfactory.